Initial Conditions and the Structure of the Singularity in Pre-Big-Bang Cosmology

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Abstract

We propose a picture, within the pre-big-bang approach, in which the universe emerges from a bath of *plane* gravitational and dilatonic waves. The waves interact gravitationally breaking the exact plane symmetry and lead generically to gravitational collapse resulting in a singularity with the Kasner-like structure. The analytic relations between the Kasner exponents and the initial data are explicitly evaluated and it is shown that pre-big-bang inflation may occur within a dense set of initial data. Finally, we argue that plane waves carry zero gravitational entropy and thus are, from a thermodynamical point of view, good candidates for the universe to emerge from.

1 Introduction

The low energy effective equations of string theory provide cosmological solutions which might be applicable just below the string scale in the very early universe. In the pre-big-bang (PBB) scenario, suggested naturally by the spirit and the symmetries of Superstring theory, the universe starts in a low curvature, low coupling regime and then enters a stage of dilaton driven kinetic inflation [1]. To address one of the main problems of cosmology, namely the problem of the initial conditions, this interesting picture has been developed further in [2], where the authors suggest that the initial state of the universe could have consisted of a bath of gravitational and dilatonic waves, some of which would have collapsed leading to the birth of a baby inflationary universe. These PBB bubble universes would give rise, finally, after a yet-to-be clarified graceful exit mechanism, to the observed Friedman-Robertson-Walker (FRW) world.

The main purpose of this paper is to develop a modified *realization* of the PBB bubble picture of Buonanno, Damour and Veneziano [2], in which the spherically symmetric collapse

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leading to inflationary PBB solutions is substituted by the interaction of strictly plane waves. This modification affects only the initial state of the universe, while near the (spacelike) caustic singularity the model shows similar behaviour to that discussed in [2], leading to Kasner-like structure. Representing exact solutions to the classical string equations of motion to all orders in the inverse string tension [3], it looks rather natural to modify the PBB picture by incorporating plane waves into the postulate of "asymptotic past triviality" [2]. Moreover, this picture is attractive not only due to the exactness of plane wave backgrounds for string propagation, but most importantly, because of mutual "fatal attraction" exercised by the plane waves which leads to an inevitable gravitational collapse independently of their strength, unlike in the spherical picture.

In the scenario we propose, one starts with a model universe in a low coupling, low curvature regime with plane gravitational and matter waves which eventually will gravitationally interact [1]. Colliding plane wave space-times have been investigated in detail in general relativity (see [4] and references therein). A generic feature of the interaction of two plane waves is the formation of a strong space-like curvature singularity in the future [6]. In the context of the PBB scenario, this singularity can be re-interpreted as an ordinary cosmological singularity.

The approach to the singularity from the past occurs through a Kasner-like behaviour, and this, in turn, can be analytically related to the initial data. Therefore, one is able to address in full the important question of fine tuning of initial conditions [7, 8, 9, 10] leading to inflationary behaviour as $t \to 0^-$. This is a well defined problem in our picture, again, unlike in the spherically symmetric case, where similar analysis does not seem to be possible. In the case at hand, the problem is a direct generalisation of the problem of determining Kasner exponents in scattering of pure plane gravitational waves with constant polarization [11]. As far as the technical part of this paper is concerned, some already well-known results from general relativity and mathematical cosmology will be used and re-interpreted in a new light.

The paper is organized as follows. In Sec. 2 we review briefly the initial value problem for the collision of dilatonic and gravitational waves. In particular we provide a closed expression relating the Kasner exponents which characterize the asymptotic geometry near the caustic singularity with the initial conditions for the metric functions and the dilaton. In Sec. 3 we apply these results to investigate the range of initial conditions leading to PBB inflation, and whether these conditions are naturally met in the collision of plane gravitational waves. In Sec. 4 we study two particular geometries in the interaction region, the Nappi-Witten solution [12], and a family of Kantowski-Sachs metrics studied in [10]. Finally, in Sec. 5 we will use thermodynamical considerations to argue that plane waves are good candidates to represent the primordial PBB universe, summarize our conclusions and indicate possible future directions of research.

2 Colliding plane waves with aligned polarization

One of the basic assumptions of the PBB scenario is the so-called "asymptotic past triviality" (APT) hypothesis. According to it, the universe starts in the asymptotic past in a low curvature and low string coupling regime where the physics is accurately described in terms of tree level string theory. The effective dynamics of the long-range fields is thus governed by the leading terms of the effective low energy string action where both quantum and α'

[1]In the gravitational sector we limit ourselves to the discussion of constantly polarized waves (diagonal metrics) on the grounds that in string theory the collision of gravitational waves with variable polarization may be generically mapped via T-duality into another problem where the variable polarization of the incoming waves is transformed into an non-vanishing value of the $B_{\mu\nu}$ field [5].

corrections are ignored. In four dimensions this action is given in the string frame by [13, 14]

$$S = \int d^4x\sqrt{-g}e^{-\phi}\left(R + g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - \frac{1}{12}H^{\alpha\beta\gamma}H_{\alpha\beta\gamma}\right) \tag{1}$$

where the dilaton ϕ and the antisymmetric tensor field strength $H_{\alpha\beta\gamma} = \partial_{[\alpha}B_{\beta\gamma]}$ are introduced. Furthermore, we will assume throughout that the extra six spatial dimensions are compactified in some internal appropriate manifold considered to be non-dynamical.

Applying the conformal transformation

$$g_{\alpha\beta} \rightarrow e^{-\phi}g_{\alpha\beta}. \tag{2}$$

the action can be written in the usual Einstein-Hilbert form (Einstein frame). In this frame the equations of motion are given by [14]

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= {}^{(\phi)}T_{\mu\nu} + {}^{(H)}T_{\mu\nu} && (3)\\ \nabla_\mu\left[\exp(-2\phi)H^{\mu\nu\lambda}\right] &= 0 && (4)\\ \Box\phi + \frac{1}{6}e^{-2\phi}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma} &= 0 && (5) \end{aligned}$$

where

$$\begin{aligned} {}^{(\phi)}T_{\mu\nu} &= \frac{1}{2}(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}) && (6)\\ {}^{(H)}T_{\mu\nu} &= \frac{1}{12}e^{-2\phi}\left(3H_{\mu\lambda\kappa}H_\nu{}^{\lambda\kappa} - \frac{1}{2}g_{\mu\nu}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma}\right) && (7) \end{aligned}$$

In four dimensions the antisymmetric tensor field strength can be written in terms of the pseudoscalar field, b, as follows

$$H^{\mu\nu\lambda} = e^{2\phi}\epsilon^{\rho\mu\nu\lambda}b_{,\rho}, \tag{8}$$

and since the solutions including the axion b can be obtained from pure dilaton solutions via a $SL(2, I\!R)$ transformation leaving invariant the Einstein frame metric, we will ignore this field and concentrate on gravi-dilaton system in what follows.

In [2] the following two assumptions are required to hold for an asymptotically past trivial (APT) initial state:

- APT_1: The string theory is weakly coupled, i.e. $g = e^{\phi/2} \ll 1$.
- APT_2: The curvature in string units is small.

APT_1 ensures that classical string theory is valid and string loop corrections to (1) can be ignored, whereas APT_2 means that α' corrections to the same action are negligible. Our starting point will be to assume that in the asymptotic past the universe is in a trivial state characterized by gravitational waves propagating in a flat space-time. Eventually, these plane waves will collide giving rise to non-trivial geometries in the interaction region, and in particular, possibly, to the nucleation of PBB universes. Since plane waves are exact string vacua, APT_2 is automatically satisfied in the space-time regions before the interaction [3]. However, it is not at all clear that the exact conformal invariance of the background also holds for all the possible solutions describing the interaction region. From physical considerations one would expect to have at least one of these solutions in the interaction region corresponding to an exact string background, and that this solution would smoothly match the incoming plane waves along the null boundaries of the interaction region (see below).

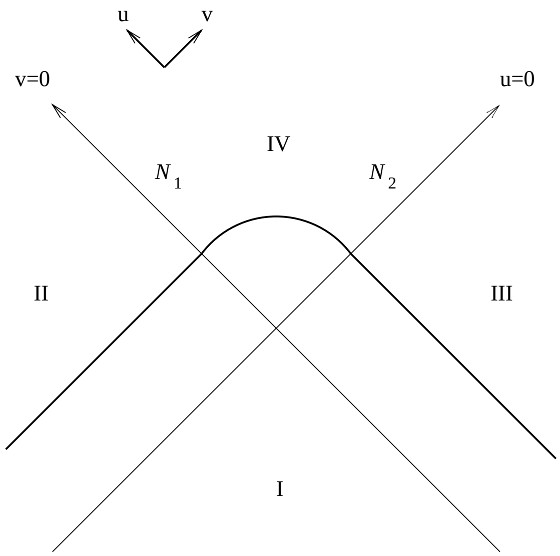

Figure 1: Region I is the flat background space-time, region II and III describe the approaching plane waves and region IV is the interaction region.

The space-times representing interactions of plane waves have two commuting Killing vectors ζ_1, ζ_2 and it is possible to chose a system of adapted coordinates (u, v, x, y) in which $\zeta_1 = \partial_x$ and $\zeta_2 = \partial_y$, whereas the longitudinal coordinates (u, v) are null. In the u-v plane, the resulting space-time can be divided into four different regions (cf. Fig. 1) [15, 16, 4]:

- Region I ($u < 0$, $v < 0$) is flat space-time, described by the usual Minkowski line element
$$ds_I^2 = -2dudv + dx^2 + dy^2$$
and a constant dilaton.
- Region II ($u \geq 0$, $v \leq 0$) is incoming plane wave 1, described by
$$ds_{II}^2 = -2dudv + F_1^2(u)dx^2 + G_1^2(u)dy^2, \tag{9}$$
and dilaton field $\phi_1(u)$.
- Region III ($u \leq 0$, $v \geq 0$) is incoming plane wave 2, described by
$$ds_{III}^2 = -2dudv + F_2^2(v)dx^2 + G_2^2(v)dy^2, \tag{10}$$
along with $\phi_2(v)$.
- Region IV ($u \geq 0$, $v \geq 0$) is the interaction region, described by
$$ds_{IV}^2 = -2e^{-F}dudv + G(e^{\psi}dx^2 + e^{-\psi}dy^2), \tag{11}$$
where $F(u, v)$, $G(u, v)$ and $\psi(u, v)$, as well as the dilaton field $\phi(u, v)$, are functions of both u and v.

In this problem the initial data are most conveniently posed on the null surfaces $N_1 \cup N_2$, where $N_1 \equiv \{v = 0, u \geq 0\}$ and $N_2 \equiv \{u = 0, v \geq 0\}$, which is the boundary of the interaction region IV. In the interior of this region one of Einstein's equations reads
$$G_{uv} = 0 \tag{12}$$

which is solved by [16, 17, 4]

$$G = a(u) + b(v) = 1 - (\alpha u)^n - (\beta v)^m.$$

Here α^{-1} and β^{-1} are arbitrary positive length scales fixing the focal lengths of the incoming waves that we will set to 1 in the following. On the other hand, the integers n and m are determined by the boundary conditions.

Introducing two new coordinates ξ and z defined by ($\alpha = \beta = 1$)

$$\begin{aligned} \xi &\equiv a(u) + b(v) = 1 - u^n - v^m &(13)\\ z &\equiv a(u) - b(v) = u^n - v^m &(14) \end{aligned}$$

the metric (11) in the interaction region takes the familiar Einstein-Rosen form

$$ds^2 = e^f(-d\xi^2 + dz^2) + \xi(e^\psi dx^2 + e^{-\psi} dy^2). \qquad (15)$$

The equations of motion for the metric functions and the dilaton are given by

$$\begin{aligned} \ddot{\psi} &+ \frac{1}{\xi}\dot{\psi} - \psi'' = 0 &(16)\\ \ddot{\phi} &+ \frac{1}{\xi}\dot{\phi} - \phi'' = 0 &(17)\\ \dot{f} &= -\frac{1}{2\xi} + \frac{\xi}{2}\left(\dot{\psi}^2 + \psi'^2\right) + \frac{\xi}{2}\left(\dot{\phi}^2 + \phi'^2\right) &(18)\\ f' &= \xi\dot{\psi}\psi' + \xi\dot{\phi}\phi' &(19) \end{aligned}$$

where a dot and a prime denote differentiation with respect to ξ and z respectively. The equations for ψ and ϕ can be solved in terms of Bessel and Neumann functions by [17]

$$V = k\ln\xi + \mathcal{L}\{A_\omega \cos[\omega(z+z_0)]J_0(\omega\xi)\} + \mathcal{L}\{B_\omega \cos[\omega(z+z_0)]N_0(\omega\xi)\}\,. \qquad (20)$$

Here V stands for either ψ or ϕ and $\mathcal{L}\{...\}$ denotes arbitrary linear combinations of the terms in curly brackets including those of the form $\int_0^\infty A_\omega \cos[\omega(z+z_0)]J_0(\omega\xi)$, $\int_0^\infty B_\omega \cos[\omega(z+z_0)]N_0(\omega\xi)$.

In order to relate the asymptotic behaviour near the singularity at $\xi = 0$ to the initial data given on the boundary of the interaction region at $\{(u,0)\} \cup \{(0,v)\}$ it is useful to introduce yet another set of coordinates r and s defined by

$$r \equiv \xi - z, \qquad s \equiv \xi + z.$$

In this case the equations for ψ and ϕ take the form

$$\begin{aligned} \psi_{,rs} + \frac{1}{2(r+s)}(\psi_{,r} + \psi_{,s}) &= 0 &(21)\\ \phi_{,rs} + \frac{1}{2(r+s)}(\phi_{,r} + \phi_{,s}) &= 0. &(22) \end{aligned}$$

These two equations, together with the initial data on the null boundaries of the interaction region $\{\psi(r,1), \psi(1,s)\}$ and $\{\phi(r,1), \phi(1,s)\}$ pose a well defined initial value problem. Both $\psi(r,s)$ and $\phi(r,s)$ are C^1 (and piecewise C^2) functions. This problem was first solved by Szekeres [16] in the case of pure gravitational waves. Here the notation of Yurtsever [11] is used,

$$\psi(r,s) = \int_1^s ds' \left[\psi_{,s'}(1,s') + \frac{\psi(1,s')}{2(1+s')}\right]\left[\frac{1+s'}{r+s}\right]^{\frac{1}{2}} \mathcal{P}_{-\frac{1}{2}}\left[1 + 2\frac{(1-r)(s'-s)}{(1+s')(r+s)}\right]$$

$$+\int_1^r dr' \left[\psi_{,r'}(r',1)+\frac{\psi(r',1)}{2(1+r')}\right]\left[\frac{1+r'}{r+s}\right]^{\frac{1}{2}} \mathcal{P}_{-\frac{1}{2}}\left[1+2\frac{(1-s)(r'-r)}{(1+r')(r+s)}\right] \quad (23)$$

$$\phi(r,s) = \int_1^s ds' \left[\phi_{,s'}(1,s')+\frac{\phi(1,s')}{2(1+s')}\right]\left[\frac{1+s'}{r+s}\right]^{\frac{1}{2}} \mathcal{P}_{-\frac{1}{2}}\left[1+2\frac{(1-r)(s'-s)}{(1+s')(r+s)}\right]$$
$$+\int_1^r dr' \left[\phi_{,r'}(r',1)+\frac{\phi(r',1)}{2(1+r')}\right]\left[\frac{1+r'}{r+s}\right]^{\frac{1}{2}} \mathcal{P}_{-\frac{1}{2}}\left[1+2\frac{(1-s)(r'-r)}{(1+r')(r+s)}\right] \quad (24)$$

where $\mathcal{P}_{-\frac{1}{2}}(x)$ is a Legendre function. It is important to stress here that, in order to study the behaviour near the singularity, we are only concerned with the "nonzero mode" of the dilaton. Therefore, $\phi(r,s)$ in Eq. (24) is normalized in such a way that $\phi(1,1)=0$. However, we can always add an arbitrary constant to the dilaton field and still have a solution to the wave equation (22). In particular, we can tune the string coupling constant to small values in regions II and III (and of course in I as well) without affecting the structure of the singularity in region IV.

Near the singularity at $\xi=0$ Kasner behaviour is expected, and it is known that space-times admitting two abelian space-like Killing vectors with parallel polarization have an asymptotically velocity dominated singularity [18] so that curvature effects become negligible there.

The nice feature of the colliding plane wave space-times is that the initial value problem is well posed and can be solved exactly. This allows to relate the Kasner exponents which describe the behaviour of the metric near the singularity to the initial data given on the null boundaries of the interaction region. In order to find this relationship we expand the Bessel functions around $\xi=0$, and proceeding along the lines of Yurtsever's work [11] write the following decomposition

$$\psi(\xi,z) = \epsilon(z)\ln\xi + d(z) + H(\xi,z),$$
$$\phi(\xi,z) = \varphi(z)\ln\xi + \tilde{d}(z) + \tilde{H}(\xi,z),$$

where $\epsilon(z)$, $\varphi(z)$, $d(z)$ and $\tilde{d}(z)$ are independent of ξ, and $H(\xi,z)$, $\tilde{H}(\xi,z)$ vanish in the limit $\xi\to 0$ [or $r+s\to 0$ in (r,s) coordinates]. Thus, the Kasner exponents in this limit are determined entirely by the coefficients of the logarithmic terms in the above expansions.

These coefficients can be computed directly from Eqs. (23) and (24). For $\epsilon(z)$ one finds

$$\epsilon(z) = \frac{1}{\pi\sqrt{1+z}}\int_z^1 ds \left[(1+s)^{\frac{1}{2}}\psi(1,s)\right]_{,s}\left(\frac{s+1}{s-z}\right)^{\frac{1}{2}}$$
$$+ \frac{1}{\pi\sqrt{1-z}}\int_{-z}^1 dr \left[(1+r)^{\frac{1}{2}}\psi(r,1)\right]_{,r}\left(\frac{r+1}{r+z}\right)^{\frac{1}{2}}, \quad (25)$$

and a similar expression holds for the dilaton

$$\varphi(z) = \frac{1}{\pi\sqrt{1+z}}\int_z^1 ds \left[(1+s)^{\frac{1}{2}}\phi(1,s)\right]_{,s}\left(\frac{s+1}{s-z}\right)^{\frac{1}{2}}$$
$$+ \frac{1}{\pi\sqrt{1-z}}\int_{-z}^1 dr \left[(1+r)^{\frac{1}{2}}\phi(r,1)\right]_{,r}\left(\frac{r+1}{r+z}\right)^{\frac{1}{2}}. \quad (26)$$

By introducing the leading logarithmic behaviour of both $\psi(\xi,z)$ and $\phi(\xi,z)$ into the equations for $f(\xi,z)$, Eqs. (18) and (19), we readily get the solution for the metric function $f(\xi,z)$ near the singularity at $\xi=0$ to be

$$f(\xi,z) \simeq \frac{1}{2}[\epsilon^2(z)+\varphi^2(z)-1]\ln\xi. \quad (27)$$

Hence the asymptotic behaviour of the metric when $\xi \to 0$ is given by

$$ds^2 = \xi^{a(z)}(-d\xi^2 + dz^2) + \xi^{1+\epsilon(z)}dx^2 + \xi^{1-\epsilon(z)}dy^2 \tag{28}$$

where $a(z) \equiv \frac{1}{2}[\epsilon^2(z) + \varphi^2(z) - 1]$ (cf. also [19]).

Thus, we have completely specified the asymptotic behaviour of the metric near the caustic singularity in terms of the initial data for $\psi(\xi, z)$ and $\phi(\xi, z)$, as encoded by the functions $\epsilon(z)$ and $\varphi(z)$. In the following section we will use this result to study the initial conditions in the collision problem leading to PBB inflation.

3 Conditions for pre-big-bang inflation

Now that the relation between the asymptotic form of the metric (28) and the initial conditions on the boundary of the interaction region is given, we can address the problem of determining what kind of initial data lead to PBB inflationary solutions. Transforming the solution (28) to the string frame and switching, once in the string frame, from conformal to cosmic time we find the following Kasner exponents (generically, these will be functions of z)

$$\begin{aligned} p_1(z) &\equiv \frac{1+\epsilon(z)+\varphi(z)}{b(z)+2} \\ p_2(z) &\equiv \frac{1-\epsilon(z)+\varphi(z)}{b(z)+2} \\ p_3(z) &\equiv \frac{b(z)}{b(z)+2} \end{aligned} \tag{29}$$

where $b(z) \equiv \frac{1}{2}[\epsilon^2(z) + \varphi^2(z) + 2\varphi(z) - 1]$ and the subscripts 1,2,3 correspond to the x, y, z directions respectively. These exponents satisfy the usual conditions for dilaton-vacuum Kasner solutions in the string frame

$$\sum_{i=1}^{3} p_i(z) = 1 + \frac{2\varphi(z)}{b(z)+2}, \qquad \sum_{i=1}^{3} p_i(z)^2 = 1 .$$

The conditions for PBB inflation ($p_1, p_2, p_3 < 0$) are then translated into the following conditions on the functions $b(z)$, $\epsilon(z)$ and $\varphi(z)$:

$$-2 < b(z) < 0 \tag{30}$$

$$1 - \epsilon(z) + \varphi(z) < 0 \tag{31}$$

$$1 + \epsilon(z) + \varphi(z) < 0 . \tag{32}$$

Actually, the first inequality, when expressed in terms of $\epsilon(z)$ and $\varphi(z)$, reads

$$\epsilon(z)^2 + [\varphi(z) + 1]^2 < 2 \tag{33}$$

Thus, the set of points in the $\epsilon(z)$-$\varphi(z)$ plane for which we get PBB inflationary solutions near the singularity is the quadrant of the circle defined by (33) and bounded by the lines (31) and (32) as shown in Fig. 2. This quadrant is inscribed on the square defined by $|\epsilon(z)| < \sqrt{2}$ and $|\varphi(z) + 1| < \sqrt{2}$.

Since for the inflating models $\varphi(z) < 0$,

$$g_{\text{eff}} \sim (-\xi)^{\frac{1}{2}\varphi(z)} \longrightarrow +\infty, \qquad (\xi \to 0^-)$$

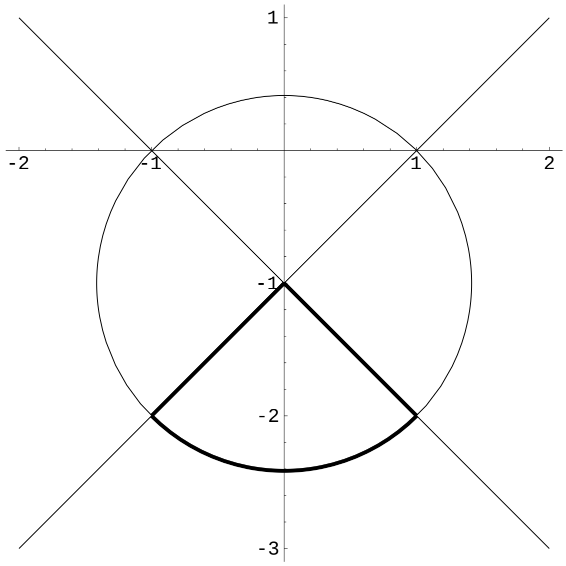

Figure 2: Parameter space on the $\varphi(z)$ vs. $\epsilon(z)$ plane. The thick line bounds the region representing those models for which inflation occurs in all directions.

so the effective string coupling constant diverges at the singularity. As a consequence, near $\xi = 0$ quantum corrections will be large and will lead, hopefully, to regularization of the singularity and a graceful exit into the post-big-bang (i.e. FRW) phase.

Now the question of genericity of PBB inflation can be posed. In order to answer this we impose APT on the initial data. APT_1 demands that the string coupling constant has to be small. Since our equations are invariant under the shift of the dilaton by a constant $\phi \to \phi+$constant we can always fix this constant in such a way that the string theory is weakly coupled in regions I, II and III and thus also at the null boundaries $N_1 = \{v = 0, u > 0\}$, $N_2 = \{u = 0, v > 0\}$.

On the other hand, APT_2 is implemented by requiring small curvatures. We can mathematically express this condition by demanding the components of the Weyl tensor to be small on the initial null hypersurfaces N_1 and N_2. On N_1 the only non-vanishing component is Ψ_4 and it is given by

$$\Psi_4\big|_{N_1} = -\frac{1}{2}\psi_{uu} + \frac{1}{4}\psi_u \left[2\frac{n-1}{u} + \frac{3n\,u^{n-1}}{1-u^n} - \frac{1-u^n}{n\,u^{n-1}}(\psi_u^2 + \phi_u^2)\right] \tag{34}$$

where we have used the notation $\psi_1(u) \equiv \psi(u, v = 0)$ and $\phi_1(u) \equiv \phi(u, v = 0)$. On the other hand, on N_2 only Ψ_0 is non-zero, and we find

$$\Psi_0\big|_{N_2} = -\frac{1}{2}\psi_{vv} + \frac{1}{4}\psi_v \left[2\frac{m-1}{v} + \frac{3m\,v^{m-1}}{1-v^m} - \frac{1-v^m}{m\,v^{m-1}}(\psi_v^2 + \phi_v^2)\right] \tag{35}$$

where now[2] $\psi_2(v) \equiv \psi(u = 0, v)$ and $\phi_2(v) \equiv \phi(u = 0, v)$.

Thus, in order to satisfy generically the conditions of having low curvature in string units, $\Psi_4|_{N_1}, \Psi_0|_{N_2} \ll 1$, we have to demand that all the derivatives of the metric functions $\psi_1(u)$,

[2] In writing Ψ_4 and Ψ_0 we have set the original focal lengths of the incoming waves α^{-1}, β^{-1} to 1. We can restore these length scales by writing $\Psi_4|_{N_1} \to \alpha^{-2}\Psi_4|_{N_1}$ and $\Psi_0|_{N_2} \to \beta^{-2}\Psi_0|_{N_2}$ in Eqs. (34) and (35) respectively. Since the string length ℓ_{st} is the natural scale of the problem, we take $\alpha^{-1} = \beta^{-1} = \ell_{st} = 1$ and measure all curvatures in string units.

$\psi_2(v)$ that appear in expressions (34) and (35), as well as the corresponding derivatives of the dilaton field on the boundaries, $\phi_1(u)$, $\phi_2(v)$ are much smaller than 1. From the condition $\psi(u=0, v=0) = 0$ and $\phi(u=0, v=0) = 0$ [in the latter case by $\phi(u,v)$ we denote just the "nonzero mode" of the dilaton] and the smallness of the derivatives we conclude that the functions $\psi_1(u)$, $\psi_2(v)$ as well as $\phi_1(u)$ and $\phi_2(v)$ are approximately constant and close to zero. Consequently, the initial data compatible with APT will satisfy

$$\psi(1, s) \simeq \mu_1 = \text{constant} \ll 1, \qquad \psi(r, 1) \simeq \mu_2 = \text{constant} \ll 1, \tag{36}$$

and similar relation for the "nonzero mode" of the dilaton field

$$\phi(1, s) \simeq \nu_1, \qquad \phi(r, 1) \simeq \nu_2 \tag{37}$$

where again ν_1 and ν_2 are constants much smaller than 1. If we now make use of the expressions (25) and (26) that give us the gravitational and matter source functions $\epsilon(z)$ and $\varphi(z)$ in terms of the initial data, we find that $\epsilon(z)$ is given by

$$\epsilon(z) \simeq \left(\frac{1-z}{1+z}\right)^{\frac{1}{2}} \frac{\mu_1}{\pi} + \left(\frac{1+z}{1-z}\right)^{\frac{1}{2}} \frac{\mu_2}{\pi}. \tag{38}$$

This expression implies that $\epsilon(z) \ll 1$ for a large range of values of $z \in (-1, 1)$, as long as $\mu_1, \mu_2 \ll 1$. On the other hand for the dilaton we get a similar relation

$$\varphi(z) \simeq \left(\frac{1-z}{1+z}\right)^{\frac{1}{2}} \frac{\nu_1}{\pi} + \left(\frac{1+z}{1-z}\right)^{\frac{1}{2}} \frac{\nu_2}{\pi} \tag{39}$$

and again, since $\nu_1, \nu_2 \ll 1$, $\varphi(z) \ll 1$ for a large range of values of z.

Thus, we have found that, on general grounds, APT selects the values of $\epsilon(z)$-$\varphi(z)$ in a region around the origin $\epsilon(z) = \varphi(z) = 0$. If for $\epsilon(z)$ this is consistent with the coordinates of those points in parameter space corresponding to the models for which the nucleation of PBB bubbles happens (see Fig. 2), in the case of $\varphi(z)$ the situation is not that good, since to achieve PBB inflation we need $-1-\sqrt{2} < \varphi(z) < -1$. In any case, it is important to notice that the bounds imposed by APT are not equally strong for $\psi(u,v)$ and $\phi(u,v)$. While in order to fulfill APT_2 we need both the first and second derivatives of $\psi(u,v)$ to be much smaller than 1 on N_1 and N_2, for $\phi(u,v)$ we need just to demand this same condition on the *square* of the first derivative. Thus, APT is compatible with the hierarchy between the constants μ_1, μ_2 and ν_1, ν_2.

Nevertheless, the important conclusion we have reached is that, as a result of the gravitational wave collision, PBB inflation happens for a dense set of initial data, i.e. the PBB inflation becomes "a piece of cake" in our scenario. This means that, once we have an inflationary solution, inflation is stable under small perturbations of the initial conditions that lead to small variations of the Kasner exponents.

4 Particular solutions in the interaction region

In the previous Section we have discussed the general asymptotic behaviour of the solutions near the curvature singularity and have shown that one may completely specify the structure of the singularity in terms of initial data posed on the null boundaries of the interaction region. In what follows, we discuss two particular examples of metrics in this region.

It is known that any metric with two commuting spatial Killing directions describes the interaction region in a colliding wave problem provided the appropriate boundary conditions are met. Here we will concentrate our attention on two cases which we think are of physical relevance. First, we will study, in the light of our approach, the solution of Nappi and Witten which describe an inhomogeneous universe with closed spatial sections of S^3 topology [12]. The most interesting feature of this solution is the fact that it is an exact string background. After that we will consider dilatonic generalizations of the Schwarzschild metric, in order to make contact with the spherical collapse picture of Buonanno, Damour and Veneziano [2] and the PBB inflating Kantowski-Sachs universes [10].

4.1 The Nappi-Witten cosmological solution

The four-dimensional cosmological model studied by Nappi and Witten [12] results as the target space theory of a $SL(2,I\!R) \times SU(2)/SO(1,1) \times U(1)$ gauged Wess-Zumino-Witten model. The solution contains, besides the metric, non-trivial values for the dilaton and antisymmetric tensor field. Actually, the solution containing the non-vanishing B-field can be obtained by an $O(2,2;I\!R)$ rotation of the metric [20, 21] (for a review see [22])

$$ds^2 = -dt^2 + ds^2 + \tan^2 s\, dx^2 + \cot^2 t\, dy^2, \tag{40}$$

together with the dilaton field

$$\phi = \phi_0 - \log(\sin^2 t\, \cos^2 s). \tag{41}$$

The above line element may be thought of as a product of two two-dimensional black holes with Euclidean and Lorentzian signatures, both being exact string backgrounds [23], corresponding to a $SL(2,I\!R)/SO(1,1) \times SU(2)/U(1)$ coset model.

In the Einstein frame (40) is given by

$$ds^2 = e^{f(t,s)}(-dt^2 + ds^2) + K(t,s)[e^{\psi(t,s)}dx^2 + e^{-\psi(t,s)}dy^2] \tag{42}$$

with

$$\begin{aligned} f(t,s) &= \log(1-\cos 2t) + \log(1+\cos 2s) \\ K(t,s) &= \sin 2t\, \sin 2s \\ \psi(t,s) &= \log\tan t + \log\tan s, \end{aligned} \tag{43}$$

the dilaton field being given by (41). To relate the Nappi-Witten solution with the plane wave collision problem, it is convenient to switch to a different set of coordinates, namely

$$\xi = \sin 2t\, \sin 2s, \qquad z = \cos 2t\, \cos 2s \tag{44}$$

in which the solitonic nature[3] of the solutions can be made explicit. Bakas [27] also studied these solutions by applying the inverse scattering transform technique on a Kasner seed metric in a search to relate the Geroch group to the symmetries of the string theory. The above coordinate transformation can be inverted to give

$$\begin{aligned} \log\tan t &= \frac{1}{2}\left(\operatorname{arc\,cosh}\frac{1+z}{\xi} - \operatorname{arc\,cosh}\frac{1-z}{\xi}\right), \\ \log\tan s &= \frac{1}{2}\left(\operatorname{arc\,cosh}\frac{1+z}{\xi} + \operatorname{arc\,cosh}\frac{1-z}{\xi}\right) \end{aligned} \tag{45}$$

so in the new coordinates the metric function $\psi(\xi,z)$ is given by

$$\psi(\xi,z) = \operatorname{arc\,cosh}\left(\frac{1+z}{\xi}\right)$$

[3]The name solitons is owed to the inverse scattering technique used to obtain these solutions [24] rather than to their physical properties. It was realised later that in the diagonal case the gravi-soliton solutions are related to the well known Lamb-Rosen pulses [25], see [26] for a review. Their role in the colliding wave problem was discussed in [17].

whereas the dilaton becomes

$$\phi(\xi, z) = \operatorname{arc}\cosh\left(\frac{1-z}{\xi}\right) - \log\xi. \tag{46}$$

In [17] it was pointed out that the presence of at least two solitonic terms provide sufficient conditions for the continuity on the two different null boundaries of the region IV, if one is to interpret the spacetime in terms of plane wave interaction. This is indeed the case, since there is one soliton (the arc cosh term) associated with the dilaton solution and another one in the transverse metric function $\psi(\xi, z)$, and the contribution to the boundary condition of each of those is equivalent as if there where two solitons in the gravitational sector (for a general discussion of the boundary conditions in the plane wave collision problem see Ref. [4], in the string theory context see [28])

We can now express both $\psi(\xi, z)$ and $\phi(\xi, z)$ in (r, s)-coordinates ($r = \xi - z$, $s = \xi + z$), so the initial data for our problem on the boundaries N_1 and N_2 are specified by the functions

$$\psi(1, s) = 0, \qquad \psi(r, 1) = \log\left[\frac{3 - r + 2\sqrt{2(1-r)}}{1+r}\right]. \tag{47}$$

For the dilaton, on the other hand, we find

$$\phi(1, s) = -2\log\left(1 - \sqrt{\frac{1-s}{2}}\right), \qquad \phi(r, 1) = -\log\left(\frac{1+r}{2}\right).$$

By substituting these expressions into (25) and (26) we may directly study the outcome of the collision near the singularity. We find that

$$\epsilon(z) = -1, \qquad \varphi(z) = -2.$$

It can be easily seen that these values of $\epsilon(z)$ and $\varphi(z)$ lie just on the boundary of the region of points for which the model undergoes PBB inflation. If we compute the Kasner exponents using (29) we find that

$$p_1 = -1, \qquad p_2 = p_3 = 0,$$

so there is only one inflating direction, while the other two are "frozen". Incidentally, the metric near the singularity corresponds to the T-dual of Milne space-time.

The original Nappi-Witten metric [12] is obtained from the above solution by the $O(2,2;\mathbb{R})$ rotation in string frame (followed by a rescaling of the x coordinate, $x \to Bx$, see [22]). Taking into account that $O(2,2;\mathbb{R}) \sim SL(2,\mathbb{R})_\tau \times SL(2,\mathbb{R})_\rho$ the required transformation can be written as

$$\tau' = \tau, \qquad \rho' = \frac{-1}{\rho + B}, \qquad B \neq 0, \tag{48}$$

where τ and ρ are the usual Kähler and complex structure moduli constructed from the string frame metric (see, for example, [29]). As discussed in [29] the Einstein frame metric function $\psi(\xi, z)$ remains invariant under the $O(2,2;\mathbb{R})$ rotation so we have

$$\psi(\xi, z)_{\mathrm{NW}} = \operatorname{arc}\cosh\left(\frac{1+z}{\xi}\right),$$

while for the new dilaton we find

$$\phi(\xi, z)_{\mathrm{NW}} = \phi(\xi, z) - \log\left(B^2 + \xi^2 e^{2\phi(\xi, z)}\right)$$

with $\phi(\xi, z)$ given by (46). In addition, we have a non-vanishing value for the B-field

$$B_{xy}(\xi, z) = -\frac{B}{B^2 + \xi^2 e^{2\phi(\xi,z)}}.$$

Since $\psi(\xi, z)$ is left unchanged by the rotation, the initial conditions for $\psi(r, s)_{\rm NW}$ on the null boundaries N_1, N_2 are again given by (47). Therefore, the gravitational source function $\epsilon(z)$ remains invariant. On the other hand, the dilaton does transform under (48), so the initial conditions for the transformed dilaton are[4]

$$\begin{aligned} \phi(1, s)_{\rm NW} &= -\log\left(\frac{1+r}{2}\right), & (49)\\ \phi(r, 1)_{\rm NW} &= -\log\left[\frac{3-s}{2} + \frac{1-B^2}{1+B^2}\sqrt{2(1-s)}\right]. & (50) \end{aligned}$$

Using Eq. (26) we can check that the scalar source function $\varphi(z)_{\rm NW}$ vanishes. Consequently, the model lies outside the inflationary region in the $\epsilon(z)$-$\varphi(z)$ plane. If we evaluate the related Kasner exponents using (29) we get

$$p_2 = 1, \qquad p_1 = p_3 = 0,$$

so the metric asymptotically approaches the Milne regime as $\xi \to 0^-$.

In fact, we may perform a somewhat more general analysis; if we start with a solution characterized by some values of $\epsilon(z)$, $\varphi(z)$ within the inflationary region, after a generic $SL(2, I\!R)_\rho \subset O(2, 2; I\!R)$ rotation the resulting metric near the singularity will be characterized by the new functions

$$\bar{\epsilon}(z) = \epsilon(z), \qquad \bar{\varphi}(z) = -\varphi(z) - 2. \tag{51}$$

In particular, for every model leading to PBB inflation we have $-\sqrt{2}-1 < \varphi(z) < -1$, so the transformed function $\bar{\varphi}(z)$ will satisfy $\bar{\varphi}(z) > -1$ and thus the metric will not inflate at the singularity. Since transformations in the $SL(2, I\!R)_\rho$ factor of $O(2, 2; I\!R)$ are the ones generating background values of the B-field, one might be tempted to conclude that PBB inflation is not robust under the introduction of this field. On the other hand, some of the models which were not inflating before the transformation was performed, may happen to inflate after. Incidentally, all points in Fig. 2 with $\varphi(z) \geq -1$ are preserved by transformations

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in SL(2, I\!R)_\rho$$

with $D \neq 0$, whereas they transform as in (51) when $D = 0$.

4.2 Dilatonic Schwarzschild-like metric

In the original picture of Ref. [2], the nucleation of PBB bubbles comes through gravitational instability in the asymptotically trivial Universe. In our proposal, on the other hand, this nucleation is not so much due to gravitational instability of the gravitational wave gas, but rather, the result of the mutual focusing of these waves due to their nonlinear interaction. Needless to say that the initial conditions are specified by quite different initial data in both pictures. What we propose here is the closest thing one might think to represent the

[4]Notice again that in giving the initial conditions for the dilaton we are restricting to the "nonzero mode" defined by $\phi(1, 1) = 0$ in (r, s)-coordinates. This means in particular that in the case of the Nappi-Witten solution we should write the arbitrary additive constant ϕ_0 in the dilaton as $\phi_0 + \log(1 + B^2)$ in order to recover this nonzero mode when $\phi_0 = 0$.

decomposition of the initial data into plane waves in a non-linear theory. In what follows we will consider a set of initial data expressed as plane waves producing the same behaviour in the interaction region, as if it were a particular case of spherically symmetric gravitational collapse and will relate this to the solutions discussed in [2] and [10]. The indication that this is possible relies on the previous studies [30] where the solution first obtained by Ferrari and Ibáñez [31], and representing part of the black hole, were investigated in detail.

To this end, we start again with the Gowdy metric (42) specifying

$$
\begin{aligned}
f(t,s) &= \frac{1}{2}[(a+1)^2+b^2-1]\log\sin 2t - a\log(1+\cos 2t) && (52)\\
K(t,s) &= \sin 2t \sin 2s && (53)\\
\psi(t,s) &= a\log\tan t + \log\sin 2t\sin 2s && (54)
\end{aligned}
$$

and dilaton field

$$\phi(t,s) = \phi_0 + b\log\tan t$$

where the two constants a and b satisfy the condition $a^2+b^2=4$. A common feature of this uniparametric family of solutions is that they are spatially homogeneous and of Kantowski-Sachs type with positive spatial curvature[5]. In particular, for $a=2$ and $b=0$, we obtain the "inside-horizon region" of a Schwarzschild black hole with $4M^2=1$. From (52)-(54) we see that the metrics are singular at $t=0,\frac{\pi}{2}$. Whenever $b\neq 0$ these are true curvature singularities with the curvature invariants blowing up. On the other hand, when $b=0$ ($a=\pm 2$) the apparent singularity at $t=0$ is just a coordinate singularity, while the one at $t=\frac{\pi}{2}$ remains a curvature singularity.

We can now rewrite these solutions using (ξ,z)-coordinates defined by Eq. (44). Doing so the metric function $\psi(\xi,z)$ and the dilaton field $\phi(\xi,z)$ are

$$
\begin{aligned}
\psi(\xi,z) &= \frac{a}{2}\left(\operatorname{arc\,cosh}\frac{1+z}{\xi}+\operatorname{arc\,cosh}\frac{1-z}{\xi}\right)+\log\xi\\
\phi(\xi,z) &= \frac{b}{2}\left(\operatorname{arc\,cosh}\frac{1+z}{\xi}+\operatorname{arc\,cosh}\frac{1-z}{\xi}\right)
\end{aligned}
$$

Changing into (r,s)-coordinates we readily get the initial conditions for $\psi(r,s)$ on the null boundaries N_1, N_2

$$
\begin{aligned}
\psi(1,s) &= \frac{a}{2}\log\left[\frac{3-s+2\sqrt{2(1-s)}}{1+s}\right]+\log\left(\frac{1+s}{2}\right),\\
\psi(r,1) &= \frac{a}{2}\log\left[\frac{3-r+2\sqrt{2(1-r)}}{1+r}\right]+\log\left(\frac{1+r}{2}\right).
\end{aligned}
$$

For the dilaton field we find

$$\phi(1,s)=\frac{b}{2}\log\left[\frac{3-s+2\sqrt{2(1-s)}}{1+s}\right], \qquad \phi(r,1)=\frac{b}{2}\log\left[\frac{3-r+2\sqrt{2(1-r)}}{1+r}\right].$$

From these expressions we can evaluate $\epsilon(z)$ and $\varphi(z)$ to get

$$\epsilon(z)=1-a, \qquad \varphi(z)=-b$$

[5]This family of solutions corresponds to the family of closed Kantowski-Sachs cosmologies studied in [10], as can be seen by writing them in the coordinate system $\tau=\frac{1}{2}(1+\cos 2t)$, x $=y$, $\varphi=\pi+2x$ and $\theta=\pi+2s$.

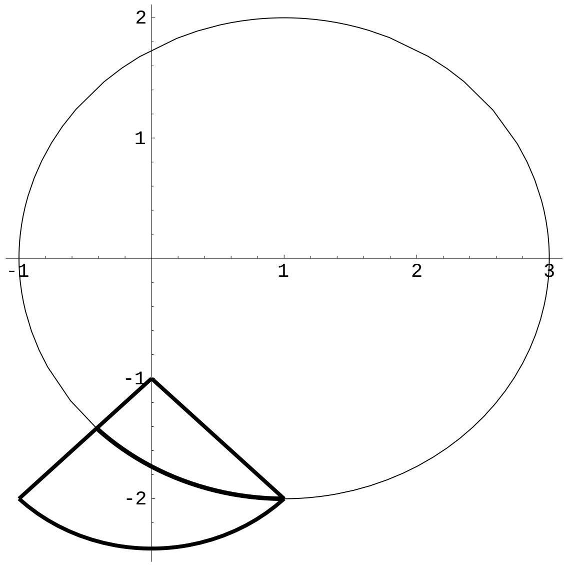

Figure 3: Points in the $\varphi(z)$ vs. $\epsilon(z)$ plane corresponding to the family of Kantowski-Sachs models. The thicker sector of the circumference represents those models for which PBB inflation occurs.

and since a and b satisfy $a^2 + b^2 = 4$, we find that the values of $\epsilon(z)$ and $\varphi(z)$ lie on the circumference defined by

$$[\epsilon(z) - 1]^2 + \varphi(z)^2 = 4. \tag{55}$$

In Fig. 3 we have plotted this curve in the $\epsilon(z)$-$\varphi(z)$ plane. We find that it crosses the region of points for which there is PBB inflation as $\xi \to 0$. Actually, the points of the circumference (55) within the inflationary region correspond to the set of models studied in [10] for which both scale factors inflate (see Fig. 2 of Ref. [10]).

We have focused our attention above to the family of deformations of the Schwarzschild black hole labeled by a single parameter and in which homogeneity is preserved, i.e. the metric is of Kantowski-Sachs type. One may construct more general deformations of the Schwarzschild metric by considering higher-dimensional moduli spaces. Moreover, the family of Kantowski-Sachs solutions studied here can be extended to a two-parametric class of solutions with "homogeneous" longitudinal part of the metric, defined by

$$\begin{aligned} f(t,s) &= \frac{1}{2}[(a_1+a_3)^2 + (b_1+b_3)^2 - 1]\log\sin 2t - (a_1a_3 + b_1b_3)\log\sin(1+\cos 2t) \\ K(t,s) &= \sin 2t \sin 2s \\ \psi(t,s) &= a_1 \log\tan t + a_2 \log\tan s + a_3 \log(\sin 2t \sin 2s) \end{aligned}$$

with the dilaton field

$$\phi(t,s) = \phi_0 + b_1 \log\tan t + b_2 \log\tan s + b_3 \log(\sin 2t \sin 2s)$$

and the set of constants $\{a_1, a_2, a_3\}$ and $\{b_1, b_2, b_3\}$ satisfying the four conditions

$$\begin{aligned} (a_1+a_2)^2 + (b_1+b_2)^2 &= 4 \\ (a_1-a_2)^2 + (b_1-b_2)^2 &= 4 \\ (a_2+a_3)^2 + (b_2+b_3)^2 &= 1 \\ a_2a_3 + b_2b_3 &= 0. \end{aligned}$$

By solving these equations and studying the behaviour of the solution close to the singularity, we find that the resulting two-parametric family of models covers the region of the $\epsilon(z)$-$\varphi(z)$ plane defined by

$$1 \leq \epsilon(z)^2 + \varphi(z)^2 \leq 9$$

which indeed contains the set of points for which PBB inflation occurs. However, if we want to relate this with the gravitational collapse picture of Ref. [2] the solutions must possess a rotational symmetry and the only models in the family with an $SO(3)$ isometry are those with $a_2 = b_2 = b_3 = 0$, $a_3 = 1$, which precisely fall into the family of Kantowski-Sachs metrics that we have studied above.

5 Conclusions and outlook

In this paper we have proposed a picture where the PBB inflation is realized starting from a trivial asymptotic state. The idea is to start with strictly plane waves moving in different directions which interact gravitationally at some stage to produce a space-time singularity. The structure of the solutions close to the singularity is of the Kasner type, and we were able to relate analytically the initial data on the wave fronts to the Kasner exponents in the string frame. Although maybe the collapsing regions studied here are not as generic as for example those studied in [2], our picture has two basic advantages:

1. The initial background is exact from the point of view of string propagation.
2. One can completely determine the structure of the singularity in terms of the initial data provided by the incoming waves.

These two elements make of this scenario, at least, a solid test bench and a sort of theoretical laboratory for the PBB ideas.

We have seen that there exists a dense set of initial data leading to inflationary behaviour in the PBB phase and, therefore, there is a good chance for an inflationary universe to emerge during this phase. Since the Kasner exponents actually carry a space dependence, i.e. they depend in general on one coordinate z, different regions with different Kasner exponents experience different types of inflation. Therefore, although we start with the collision of two plane waves we obtain near the singularity a rich structure with the formation of different multiple PBB inflationary bubbles.

Although we have extracted these conclusions from a general analysis of the collision of two gravi-dilatonic waves, we have also studied two concrete examples of initial conditions leading to different geometries in the interaction region. The first one leads to a Nappi-Witten solution in region IV and it has the obvious interest of providing an exact string background also in this region. The second example corresponds to a spherically symmetric interaction region that may describe gravitational collapse as the result of the collision.

Another motivation to look at plane wave space-times as a probable initial state for the PBB universe may come from thermodynamical considerations. One expects the universe to start in the lowest possible state of gravitational entropy. Let us suppose, arguing in the spirit of Penrose's hypothesis [32], that we relate the gravitational entropy to some homogeneous function constructed from all possible curvature invariants and that we normalize this function to be vanishing when the invariants vanish. With such a function at hand we formulate a kind of "Generalised Third Law of Thermodynamics" and assign zero gravitational entropy to those spacetimes for which *all* curvature invariants vanish. Interestingly, the FRW models do not fall into the class of zero entropy models according to our definition unlike in the Penrose's case. To justify this, we argue that the lowest entropy state, apart from being the simplest, must be an exact string background. The space-times with all vanishing curvature invariants (*plane waves*) certainly do so, while the FRW universe does not.

The additional support to consider the gravitational entropy content of the plane wave geometry to be vanishing comes from yet a different, though not unrelated argument. One would usually tend to relate the gravitational entropy with the phenomenon of quantum particle creation. It is commonly accepted that quantum particle creation indicates whether a system is endowed with nontrivial gravitational entropy. Plane waves, due to their symmetry and to the fact that all the curvature invariants vanish, do not polarize the vacuum, so quantum particles are not created in the vicinity of plane waves [33]. This is consistent then with the hypothesis of assigning zero gravitation entropy to the plane wave. Moreover, it looks as if time is not a player in the plane wave regime. Due to the absence of the global Cauchy surface, one may consider such a pure plane wave geometry as "timeless". Therefore, until two such waves interact, no notion of time as defined by entropy change is appreciated. What happens further is beyond the scope of this paper.

One of the most interesting issues, untouched here, is the problem of whether the gravitational wave collision problem can be *globally* defined in terms of an exact string background. Provided one starts with initial states that are exact string backgrounds, what are the conditions for the data to evolve into the interaction region without breaking conformal invariance? We know that one such solution exists in this region, namely the Nappi-Witten solution that we studied in Sec. 4.1. Therefore, since we start with plane gravitational waves (which are exact string backgrounds) and we can make the transition over the null boundaries as much differentiable as we like, the question remains of whether this implies conformal invariance in the interaction region. On purely physical grounds one would be tempted to say that exact string backgrounds in the regions II and III of Fig. 1 will smoothly (i.e. C^{∞}) extend to an exact background in region IV, at least if the full string equations share the uniqueness properties of the Einstein equations.

An interesting issue to address would be to refine the picture provided here to account for more realistic situations in which the primordial gravitational waves are not plane but localized. It has been argued that for "almost" plane gravitational waves singularities also occur as the result of their collision [34]. In the case of "graviton beams", there is also mutual focusing and maybe production of singularities [35] that could serve as seeds for PBB bubbles.

Finally, there has been some work on the collision of plane waves at Planckian energies [36] leading to black hole nucleation through tunneling. This kind of scenarios might be a way to find a semi-classical approximation of the formation of the singularity that could be applied to the graceful exit problem in PBB cosmology. This, and the thermodynamical ideas we have outlined above, we hope to be able to discuss in the future.

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